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$$\begin{aligned}
& +\cot^{-1}(a/d)] - \frac{1}{4}b^2[\cot^{-1}(b/c) + \cot^{-1}(b/d)] + \frac{1}{4}c^2[\tan^{-1}(a/c) - \tan^{-1}(b/c)] \\
& + \frac{1}{4}d^2[\tan^{-1}(a/d) - \tan^{-1}(b/d)] - \frac{1}{4}a(c+d) + \frac{1}{4}b(c+d) \\
& = \frac{1}{4}a^2 \cot^{-1}\left(\frac{a^2 - cd}{ac + ad}\right) - \frac{1}{4}b^2 \cot^{-1}\left(\frac{b^2 - cd}{bc + bd}\right) + \frac{1}{4}c^2 \tan^{-1}\left(\frac{ac - bc}{ab + c^2}\right) \\
& + \frac{1}{4}d^2 \tan^{-1}\left(\frac{ad - bd}{ab + d^2}\right) - \frac{1}{4}(a-b)(c+d).
\end{aligned}$$

(b) Let  $r = 4a \cos \theta / \sin^2 \theta \dots (1)$ ,  $r = 4b \cos \theta / \sin^2 \theta \dots (2)$ ,  $r = 4c \sin \theta / \cos^2 \theta \dots (3)$ ,  $r = 4d \sin \theta / \cos^2 \theta \dots (4)$ , be the equations to the parabolas;  $a > b$ ,  $c > d$ .

Let  $\tan^{-1} \mathcal{A}'(a/c) = \theta'$ .

$$A = 8a^2 \int_{\theta'}^{\frac{1}{2}\pi} \frac{\cos^2 \theta}{\sin^4 \theta} d\theta + 8c^2 \int_0^{\theta'} \frac{\sin^2 \theta}{\cos^4 \theta} d\theta = \frac{1}{8}ac.$$

Similarly,  $B = \frac{1}{8}ad$ ,  $C = \frac{2}{3}bc$ ,  $D = \frac{1}{8}bd$ .

$$\text{Area } ABCD = A + B - C - D = \frac{1}{8}(a-b)(c+d).$$

113. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College Defiance, Ohio.

Deduce the Sylvestrian Reciprocant from  $x^4 + y^4 = 4x^2y^2$ .

Solution by the PROPOSER.

The given equation may be written  $(x^2 - y^2)^2 = 2x^2y^2$ .

$\therefore x^2 - y^2 = xy\sqrt{2} \dots (a)$ .

Put  $(y/x) = w$ ; then from (a), by dividing by  $xy$ , etc., we get  $w - w^{-1} = -\sqrt{2}$ .

$\therefore w = \frac{1}{2}(-\sqrt{2} \pm \sqrt{3}) = m \dots (b)$ .

$\therefore y = mx$ , and  $dy/dx = m \dots (c)$ .

Eliminating  $m$  from (c) by a second differentiation, we have  $d^2y/dx^2 = 0 \dots (d)$ .

Adopting Professor Sylvester's notation for reciprocants, viz:  $dy/dx = t$ ,  $d^2y/dx^2 = a \ 2!$ ,  $d^2y/dx^3 = b \ 3!$ , etc., we obtain from (d) the *first* pure Sylvestrian Reciprocant. All reciprocants *not* containing  $t = dy/dx$  are *pure*; all others are *mixed*. The first two pure reciprocants are  $a$  and  $4ac - 5b^2$ , and the first two mixed ones are  $(1 + t^2)b - 2a^2t$  and  $bt - a^2$ . See our paper on Reciprocants published in the MONTHLY of November, 1894.

Also solved by G. B. M. ZERR.

114. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

When the sun's declination was  $15^\circ$  N. his altitude was found to be  $20^\circ$ , and after an hour's interval his altitude was found to be  $31^\circ$ . Required, the latitude of the place of observation.